External Force Disturbance Rejection in Robotic Arms: An Adaptive Approach

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SUMMARY  This paper is devoted to the problem of force sensorless disturbance rejection in robot manipulators. In the proposed approach, the control system uses position sensor signals and estimated values of external forces, instead of force sensor signals. The estimation process is performed via an adaptive force estimator. Then the estimated force vector is utilized to compensate for the force disturbance effect in order to achieve a better trajectory tracking performance. The force estimation is carried out directly using no environment model. Asymptotical stability of the proposed control system is analyzed by the invariant set and Lyapunov direct method establishing an appropriate theorem. Finally, the performance of the proposed control system is verified using numerical simulation.

key words: adaptive control, robotic arm, force estimation, disturbance rejection, Lyapunov direct method, invariant set theorems

1. Introduction

Manufacturing applications for force control include a great variety of commonplace tasks, such as grinding, polishing, buffing, deburring and assembly operations [1]. The difficulty of force control of robot manipulator is caused by both the rigidity of force sensor and the condition of target environment. When the robot manipulator has the effects of high temperature, large noise, etc, the force sensor cannot be mounted on it. Moreover, the force control approaches based on the feedback of force sensor signal should take the rigidity of force sensors into account, which is very difficult in the motion control of robot manipulator [2].

In order to overcome these problems, a number of approaches have been developed. Common among them is employing observer to compensate for the disturbance [3]–[17]. Various methods are used to design these observers. For example, in [4] an observer based on $H_{\infty}$ control theory for a six-link electrohydraulic manipulator is presented. Another observer, based on Internal Model Control (IMC) for detection of misinsertion in the assembly of odd-form electronic component is designed in [5]. Some of these observers are also developed for redundant [9] and cooperative [10] manipulators.

Although the disturbance observer performs desirable model regulation and disturbance rejection, it introduces problems in stability and stability robustness of the system due to the use of feedback [13]. Many important properties of existing disturbance-observers have not been established, e.g., unbiased estimation or even global stability [14]. Also, even though in some of these approaches, properties such as global stability are proved, they are developed for special cases; for example, only for two-link manipulators [14] by Chen et al. or only for systems with Coulomb friction [16], [17], by Friedl-Park, and Tafazoli et al.

Several works have been reported for adaptive force control of manipulators (e.g., that by Whitcomb et al. [1]); however, desired force control rather than force disturbance rejection has often been the aim of control. Also, they have used force sensors.

In this paper, a new scheme is introduced to estimate and reject the external force disturbance based on using an estimation of force provided by an adaptive estimator. Global convergence of the tracking and the estimation errors in this scheme is proved by using a time-varying Lyapunov function and invariant set theorems.

In the first glance, the selected Lyapunov function may seem to be similar to that selected by Spong and Ortega [18] (which is a modified form of that selected by Slotine and Li [19]); however, some main differences exist. Firstly, in the proposed scheme, the estimated parameters are components of the external force vector (consisting of external forces and torques exerted on the system), and the inertia parameters (link masses, moments of inertia, etc) of robot are known. While in [18], the estimated parameters are components of the inertia parameter vector and it is assumed that no external force is applied to the system. Hence, the aim of control in [18] is merely trajectory control by using estimation of the inertia parameters, while in the proposed scheme, both force and position are considered in control, and enhancing the trajectory tracking results from force estimation and compensation. Secondly, in [18] the skew-symmetry property is required in the design of the control system, whereas this property is not necessary in the present work and it has no restriction on use.

The proposed scheme has some desirable properties: the knowledge of force direction is not necessary; computation or estimation of joint acceleration is not needed; the convergence of tracking error is independent of initial configuration; the scheme is not developed for a special type of rigid manipulators; and as seen in the simulations of the paper, the controller gains are not too large.

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This paper consists of four sections. Section 2 discusses the proposed adaptive control algorithm, which is based on external force disturbance estimation. Section 3 shows some simulation results to verify the performance of the proposed scheme. Finally, Sect. 4 gives some concluding remarks.

2. Force Estimator Based Controller

In this section, we assume that the motion equations of robot are given. Then we derive the associated force estimation and control laws.

Dynamic equations of an n link robot are well known as

\[ M(q)\ddot{q} + h(q, \dot{q}) = B(q)\tau_{act} + J^T(q)F_{ext} \]  

where \( q \in \mathbb{R}^n \) is the vector of joint variables, \( M(q) \) is the inertia matrix \( n \times n \), \( h(q, \dot{q}) \) is the vector \( n \times 1 \) that consists of Coriolis/centripetal and gravity torques, \( \tau_{act} \) is the actuator torque vector, \( n \times 1 \), \( F_{ext} \) is the external force (and moment) vector, \( m \times 1 \) exerted on the end-effector, and \( J(q) \) is the task space Jacobian matrix, \( m \times n \).

The term \( B(q)\tau_{act} \) may be replaced by \( \tau \), the vector of joint torques, and Eq. (1) can be rewritten in a more compact form as

\[ M(q)\ddot{q} + h(q, \dot{q}) = \tau + J^T(q)F_{ext} \]  

We assume \( \tilde{f} \) to be the estimation of \( F_{ext} \) and define the force estimation error as \( \tilde{f} = F_{ext} - \hat{f} \). Also, we assume \( q_d(t) \) to be a given twice-differentiable desired trajectory in the joint space, and \( e(t) = q_d(t) - q(t) \) defines the trajectory tracking error.

In order to obtain control and estimation laws, enforcing \( e \) and \( \tilde{f} \) tend toward zero, the following theorem is presented and proved.

**Theorem:** Consider the robot control system of Fig. 1 with dynamic Eq. (2), update law \( \dot{\hat{f}} = K_f^TJr \) and control law \( \tau = M(\ddot{q}_d + \Lambda \dot{e}) + h + (\frac{1}{2} M + K)r - J^T \hat{f} \) where \( \Lambda, K \) and \( K_f \) are constant positive definite matrices and \( r = \dot{e} + \Lambda e \). Then, a) the tracking error goes asymptotically to zero, the force estimation error is bounded, and the closed-loop system is uniformly stable, b) besides, the force estimation error converges to zero provided that \( J \) satisfies

\[ \alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(q_d)J^T(q_d)dt \leq \alpha_2 I_m \]

for all \( t_0 \) and some positive scalars \( \alpha_1, \alpha_2, \) and \( T \). Then, the closed-loop system is also globally uniformly asymptotically stable.

**Proof:** a) Substituting the control law into (2) we have

\[ M \dot{e} + \left( K + \frac{1}{2} M \right) \dot{e} + J^T \hat{f} = 0 \]

Considering the update law, \( r = \dot{e} + \Lambda e \) and (3), the state equations of the closed loop system can be presented as

\[ \begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \begin{bmatrix} -M^{-1}(K + \frac{1}{2} M)\Lambda & e \\ -\Lambda^{-1} e \end{bmatrix} \begin{bmatrix} K_f^TJr \\ 0 \end{bmatrix} \]

If one replaces \( r \) in terms of \( e \) and \( \dot{e} \), the equivalent state equations of the closed loop system are written as

\[ \begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \begin{bmatrix} 0 & -M^{-1}(K + \frac{1}{2} M)\Lambda \\ -\Lambda^{-1} e \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} K_f^TJr \\ 0 \end{bmatrix} \]

Now, a Lyapunov function can be defined in terms of the state variables of the whole system, namely \( x = [e^T, \dot{e}^T, \hat{f}^T]^T \) as

\[ V(x, t) = e^T \Lambda e + \frac{1}{2} (\dot{e} + \Lambda e)^T M(\dot{e} + \Lambda e) + \frac{1}{2} \hat{f}^T K_f \hat{f} \]

Note that \( M(q(t)) = M(q_d(t) - e(t)) = M(e, t) \) is a time-varying positive definite (pd) matrix. As mentioned in [20, p.184], it is reasonable to assume that there exist positive constants \( \mu_1 \) and \( \mu_2 \) such that

\[ 0 < \mu_1 \leq \lambda_{\text{min}}(M(q)) \leq \lambda_{\text{max}}(M(q)) \leq \mu_2, \quad \forall q \]

Now to show that \( V \) is pd and decrescent function, we define

\[ V_0 = \lambda_1 e^T e + \frac{\mu_1}{2} (\dot{e} + \Lambda e)^T (\dot{e} + \Lambda e) + \frac{1}{2} \lambda_2 \hat{f}^T \hat{f} \]

\[ V_1 = \lambda_1 e^T e + \frac{\mu_2}{2} (\dot{e} + \Lambda e)^T (\dot{e} + \Lambda e) + \frac{1}{2} \lambda_2 \hat{f}^T \hat{f} \]

where \( \lambda_1 = \lambda_{\text{min}}(\Lambda^T K), \lambda_1 = \lambda_{\text{max}}(\Lambda^T K), \lambda_2 = \lambda_{\text{min}}(K_f), \lambda_2 = \lambda_{\text{max}}(K_f) \) are positive constants based on the hypotheses of the theorem. \( V_0 \) and \( V_1 \) are nonnegative and become zero only at the origin (if \( V_0 \) = 0 or \( V_1 \) = 0 then \( e = 0, \dot{e} + \Lambda e = 0 \) and \( \hat{f} = 0 \) or equivalently \( e = 0, \dot{e} = 0, \) and \( \dot{f} = 0 \); hence, they are positive definite (pd). Also,

\[ V_0(x) \leq V(x, t) \leq V_1(x) \]
Since $V_0$ and $V_1$ are pd, there exist class-K functions $\gamma_1$, $\gamma_2$, $\eta_1$ and $\eta_2$ such that
\[
\begin{align*}
\gamma_1(||x||) &\leq V_0 \leq \gamma_2(||x||), \quad \forall x \in \mathbb{R}^p; \\
\eta_1(||x||) &\leq V_1 \leq \eta_2(||x||), \quad \forall x \in \mathbb{R}^p
\end{align*}
\] (10)

where $p$ is the length of $x = [e^T, \dot{e}^T, \ddot{f}^T]^T$. Based on (9) and (10), we can write
\[
\gamma_1(||x||) \leq V(x, t) \leq \eta_2(||x||), \quad \forall t \geq 0, \quad \forall x \in \mathbb{R}^p
\] (11)

Since $V_0(0) = 0$ and $V_1(0) = 0$, it is concluded from (9) that $V(0, t) = 0$. Thus, $V$ is pd and decrescent.

If we rewrite $V$ in terms of $e$, $r$ and $\tilde{f}$, we have
\[
V = e^T \Lambda^T K e + \frac{1}{2} r^T M r + \frac{1}{2} \tilde{f}^T K_f \tilde{f}
\] (12)

Time derivative of (12) is
\[
\dot{V} = 2e^T \Lambda^T K \dot{e} + r^T M (\dot{e} + \Lambda \dot{e}) + \frac{1}{2} r^T M r + \tilde{f}^T K_f \tilde{f}
\] (13)

Substituting $\dot{e}$ by $\dot{\tilde{q}}_d - \tilde{q}$ and then replacing $M\tilde{q}$ from (2) leads to
\[
\dot{V} = 2e^T \Lambda^T K \dot{e} + r^T (M\dot{\tilde{q}}_d + h - J^T F_{ext}) + M\Lambda \dot{e} + \frac{1}{2} M r + \tilde{f}^T K_f \tilde{f}
\] (14)

Now, substituting the control law
\[
\tau = M(\dot{\tilde{q}}_d + \Lambda \dot{e}) + h + \left(\frac{1}{2} M + K\right) r - J^T \tilde{f}
\] (15)

into (14), yields
\[
\dot{V} = 2e^T \Lambda^T K \dot{e} + (\tilde{f}^T K_f - r^T J^T \tilde{f}) - r^T K r
\] (16)

Using the update law
\[
\dot{\tilde{f}} = K_f^T J r
\] (17)

the value of the second term in (16) equals to zero. It follows that
\[
\dot{V} = 2e^T \Lambda^T K \dot{e} - r^T K r
\] (18)

and by replacing $r$, it is converted to
\[
\dot{V} = e^T \Lambda^T K \dot{e} - e^T \Lambda \dot{e}
\] (19)

Hence, $\dot{V} \leq 0$, $\forall x \in \mathbb{R}^p$, $\forall t \geq 0$. $\forall \dot{V} > 0$ is decrescent, continuous in terms of time and radially unbounded in terms of the state vector $x = [e^T, \dot{e}^T, \ddot{f}^T]^T$. Thus, based on Lyapunov direct method theorems for non-autonomous systems [21, p.107], system with the state Eq. (5) is uniformly stable.

As a result, $\tilde{f}$ is bounded. Another reasoning for boundedness of $\tilde{f}$, and also a mathematical reasoning for asymptotic convergence of position and velocity tracking error can be explained as follows:

Since $V(x, t)$ is radially unbounded
\[
\forall L, \exists N(L) : \|x\| > N \rightarrow V(x, t) > L \quad \forall t \geq t_0
\] (20)

The contrapositive of this condition is
\[
\forall L, \exists N(L) : V(x, t) \leq L \rightarrow \|x\| \leq N \quad \forall t \geq t_0
\] (21)

On the other hand, since $\dot{V} \leq 0$, it can be stated that
\[
\forall \dot{V}(x, t) \leq V(x_0, t_0) = L_0 \quad \forall t \geq t_0, \quad \forall x_0 \in \mathbb{R}^p
\] (22)

Therefore, according to (21) and (22)
\[
\exists N_0(L_0) : \|x\| \leq N_0 \quad \forall t \geq t_0
\] (23)

If we suppose that $\tilde{f}$ is unbounded, then $x$ must be unbounded, which contradicts (23). Hence, $\tilde{f}$ is bounded.

In order to prove asymptotic convergence of $e$ and $\dot{e}$, we utilize the Lemma of [20, p.177]. Let us define
\[
M_f(c) = \{x \in \mathbb{R}^p : \exists \tau \geq 0 \exists V(x, t) \leq c\}
\] (24)

The level set $L_V(c)$ is defined as the connected component of $M_f(c)$ containing $0$. Next we define
\[
A_V(c) = \{x \in L_V(c) : V(x, t) \leq c, \forall t \geq 0\}
\] (25)

If $V$ is independent of $t$, then $A_V(c)$ is the same as $L_V(c)$. Regarding to the radially unboundedness of $V$, $L_V(c)$ is bounded for each bounded $c > 0$. Let
\[
S = \{x \in L_V(c) : \exists \tau \geq 0 \exists \tilde{V}(x, t) = 0\}
\] (26)

or equivalently
\[
S = \{x \in L_V(c) : \exists \tau \geq 0 \exists e = 0 and \dot{e} = 0\}
\] (27)

Let $H$ denote the largest invariant set of (5) contained in $S$. For any $x_0$ there exists a $c > 0$ such that $x_0 \in A_V(c)$. Because:

Noting to (22), it is observed that $V(x, t) \leq L_0$ for all $x_0 \in \mathbb{R}^p$. We choose $c > \sup_{||x|| < N_0} V(x, t)$ (it is clear that $L_0 < N_0$), for any $x$ in the connected set of $[||x|| < N_0, t \in [0, \infty)]$ we have $V(x, t) < t$ for all $t$. Obviously, this connected set contains the origin. Then based on the aforementioned lemma of [20, p.177] for any $x_0$, the state goes asymptotically to $H$
\[
x_0 \in \mathbb{R}^p, \quad t_0 \geq 0 \implies \lim_{t \rightarrow \infty} d(H, x) = 0
\] (28)

Where $d(H, x)$ is the distance between $H$ and $x$, or equivalently
\[
x_0 \in \mathbb{R}^p, \quad t_0 \geq 0 \implies \lim_{t \rightarrow \infty} \epsilon = 0 and \lim_{t \rightarrow \infty} \dot{\epsilon} = 0
\] (29)

b) A proof for this part can be presented by using invariant set theorems as follows. If $V \equiv 0$, then $\tau = 0$ and $\dot{\tau} = 0$; consequently, according to (3), we have
\[
J^T \tilde{f} = 0
\] (30)

If $J^T(q)$ is full column rank or equivalently $J(q)$ is full row rank, then (30) implies that $\tilde{f} = 0$. Since the closed loop system is proved to be uniformly stable in part (a), and $V$ is
and only if row rank. According to this theorem, theorem 1 of appendix to find the condition that \(J\) is a nonsingular matrix.

Figure 2 Planar RR robot manipulator.

\[\alpha_1 I_m \leq N \leq \sigma_2 I_m\]

(32)

where positive scalars \(\sigma_1\) and \(\sigma_2\) are the minimum and maximum eigenvalues of \(N\), respectively. Notice that \(N\) depends on \(t_1\) and \(t_2\), consequently, \(\sigma_1\) and \(\sigma_2\) varies with \(t_1\) and \(t_2\). If we choose \(t_1 = t_2 = t_0 + T\), and define \(\alpha_1 = \sup_{t_0(t_0, t_0 + T)} \sigma_1(t_0, t_1 + T)\) and \(\alpha_2 = \inf_{t_0 \in [0, \infty)} \sigma_2(t_0, t_1 + T)\), then \(\alpha_1\) and \(\alpha_2\) are constant scalars and we can write for all \(t_0\)

\[\alpha_1 I_m \leq N \leq \alpha_2 I_m\]

(33)

Definition of \(N\) and (33) yield

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(t) J^T(t) dt \leq \alpha_2 I_m\]

(34)

or in terms of \(q\)

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(q) J^T(q) dt \leq \alpha_2 I_m\]

(35)

It was shown in part (a) that the tracking position and velocity errors converge to zero; as a result, the above condition will be met if the desired trajectory satisfies

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(q_d) J^T(q_d) dt \leq \alpha_2 I_m\]

(36)

Hence, we derive a condition on the desired trajectory such that all force estimations will be identified after a sufficient adaptation time.

Remark 1: For adaptive computed torque method, it has shown in [25] that the system equations can be denoted as

\[
\begin{bmatrix}
\dot{X} \\
\Phi
\end{bmatrix} = \begin{bmatrix}
AX + BW\Phi \\
-\Gamma W^T C\Phi
\end{bmatrix}
\]

(37)

where \(X\) contains the tracking trajectory errors and their filtered values, and \(\Phi\) contains the unknown robot parameters that must be estimated. It has also stated in [25] based on [23] and [24] that the above system is uniformly asymptotically stable if \(W\) satisfies the persistent excitation condition

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} W^T W dt \leq \alpha_2 I_m\]

(38)

for all \(t_0\). In (37), \(\alpha_1, \alpha_2\) and \(T\) are all positive scalars. Comparing (37) with (4), it is seen that \(J^T\) in (4) has the role of \(W\) in (37). If we replace \(W\) with \(J^T\) in (38), we have

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(q) J^T(q) dt \leq \alpha_2 I_m\]

(39)

Since we have shown in part (a) that the tracking error converges to zero, the above condition will be met if the desired trajectory satisfies

\[\alpha_1 I_m \leq \int_{t_0}^{t_0 + T} J(q_d) J^T(q_d) dt \leq \alpha_2 I_m\]

(40)

which is the same as (36). Of course, notice that our approach is not based on computed torque method and the above discussion is mentioned only for denoting the similarity of the conditions.

Remark 2: (2) can be rewritten in a more detailed form as

\[M(q) \ddot{q} + V_m(q, \dot{q}) \dot{q} + G(q) = \tau + J^T(q) F_{ext}\]

(41)

Because of passive structure of rigid robots, by suitable definition of \(V_m\) in (41), \(\frac{1}{2} M - V_m\) is skew-symmetric and as a result \(r^T (\frac{1}{2} M - V_m) r = 0\). This is known as skew-symmetry property [22], which has not been used in the aforementioned proof. However, if desirable, we can utilize this property to derive a control law without \(M\) as

\[\tau = M(q_d) + \Lambda \dot{\dot{e}} + h + (V_m + K) r - J^T \tilde{f}\]

(42)

Note that this control law is different from (15), because generally speaking \(\frac{1}{2} M = V_m\). It can be shown in a similar manner that all properties mentioned in the above theorem for the system with the control law (15), are established for the system with this control law.

Also, if external force is a constant vector, (17) can be expressed as

\[\tilde{f} = K_f^{-T} J r\]

(43)
At the same time, we can compute the generalized torques applied by the actuators by\[\tau_{\text{act}} = B^{-1}\tau.\]

Remark 3: The condition in the part (b) of the theorem depends on \(J(q_d)\). Thus, the asymptotic convergence of estimation errors depends on the shape of the desired trajectory on the time.

3. Simulation

To show the efficiency of the proposed force estimation method, some numerical simulations are performed with a two DOF planar type robot manipulator depicted in Fig. 2. The dynamics of robot manipulator in Fig. 2 can be written as [22]

\[
M(q)\ddot{q} + h(q, \dot{q}) = \tau + J^T(q)F_{\text{ext}}
\]

where \(M(q), h(q, \dot{q})\) and \(J(q)\) are given by

\[
M(q) = \begin{bmatrix}
m_1l_1^2 + m_2l_2^2 & m_2l_1l_2(l_2 + l_1\cos q_2) \\
m_2l_1l_2(l_2 + l_1\cos q_2) & m_2l_2^2 \\
\end{bmatrix}
\]

\[
h(q, \dot{q}) = \begin{bmatrix}
-m_2l_1l_2(2\dot{q}_2q_2^2 + \dot{q}_2^2)\cos q_2 \\
-m_2l_1l_2\dot{q}_2^2\sin q_2 \\
+m_1 + m_2)gl_1\cos q_1 + m_2gl_2\cos(q_1 + q_2) \\
m_2gl_2\cos(q_1 + q_2) \\
\end{bmatrix}
\]

\[
J(q) = \begin{bmatrix}
-l_1\sin q_1 - l_2\sin(q_1 + q_2) - l_2\sin(q_1 + q_2) \\
l_1\cos q_1 + l_2\cos(q_1 + q_2) \\
l_2\cos(q_1 + q_2) \\
\end{bmatrix}
\]
where $\bar{l} = l_1^2 + l_2^2 + 2l_1l_2 \cos q_2$. The numerical values of the robot parameters are $l_1 = l_2 = 1 \text{ m}, m_1 = 2.3 \text{ kg}, m_2 = 0.8 \text{ kg}$ and $g = 9.8 \text{ m/s}^2$. The desired trajectory is set to be $q_d(t) = 0.5 \begin{bmatrix} \sin 2t, \cos 2t \end{bmatrix}^T \text{ (rad)}$. The initial conditions of the robot are assumed to be $q(0) = [1, -0.5]^T \text{ (rad)}, \dot{q}(0) = [0, 0]^T \text{ (rad/s)}$. Consider $F_{\text{ext}}$ is the unknown force exerted on the end-effector with the actual value of $[50, 10]^T$. Let $\hat{f}(0) = [0, 0]^T$ and the parameters of the control system as $K = \text{diag}(8, 5), K_f = 0.001I_2$ and $\Lambda = 2I_2$ where $I_2$ is the $2 \times 2$ identity matrix.

To verify the necessity of using the force signal in the control law some simulations are done. The results plotted in Fig. 3 belong to the case in which the control system of the robot does not include force sensor or force estimator. As observed, the robot cannot track the desired trajectory. Now, if a force sensor is mounted on the end-effector, the force signal is provided and we can apply it to the control law (15) by directly replacing $\hat{f}$ with the force signal. The results of this case are depicted in Fig. 4. The tracking error tends to zero and the responses are satisfactory. Therefore, we conclude that force compensation is critical and essential for good trajectory tracking performance. However, as we mentioned in the introduction, there are certain situations in which mounting force sensor faces difficulty. So it is required to supply force amount by estimating force for these situations. For the proposed control system, if the step exter-
Fig. 6 With square external force, (a) $e$, (b) $\dot{e}$, (c) $F_x$, (d) $\tau$.

Fig. 7 With sinusoidal external force, (a) $e$, (b) $\dot{e}$, (c) $F_x$, (d) $\tau$. 
nal force $[50, 10]^T$ is applied, according to Fig. 5 the force vector is identified, and the position and velocity errors converge to zero. Also, convergence of the force and tracking errors is satisfactory fast.

To show the effectiveness of the proposed scheme in case where $F_{ext}$ has variations in time, some simulations are carried out. The results are illustrated for a biased square and a biased sinusoidal external force in Fig. 6 and Fig. 7, respectively. Again we see that the control objectives are achieved successfully. In both cases, joint torques are changed properly to provide good estimation and tracking performance and are not too large.

In all of the presented simulations, joint torques have admissible magnitudes and do not exceed feasible values.

Now, we change the period of the desired trajectory $q_d(t)$ and study the estimation and control performance of the system. With 5 times increased frequency ($q_d(t) = 0.5[\sin 10t, \cos 10t]^T$), tracking error, estimated force and joint torques are plotted in Fig. 8. This figure exhibits that the estimation process are not performed as good as the estimation process of the system with $q_d(t) = 0.5[\sin 2t, \cos 2t]^T$. Also, joint torques have larger magnitudes. If the sample time is decreased to 0.5 ms, according to Fig. 9, the force is properly estimated. Thus, with suitable
sample time the estimator can appropriately estimate the external force and increasing the frequency of the desired trajectory does not degrade the performance of the whole system.

The results of this section illustrate that the proposed adaptation scheme works satisfactorily and effectively in control.

4. Conclusions

In this paper, a scheme has been introduced for adaptive control of robot in the presence of external force disturbance. This scheme is useful in situations having difficulty mounting force sensor or requiring the elimination of expensive force/torque sensor to reduce the total price. Global convergence of the tracking error and boundedness of the estimation error were proved through a theorem. At the same time, global convergence of the estimation error was proved through the same theorem under a condition. In addition, the global stability of the proposed estimator-based control system was guaranteed.

A number of simulations were presented for some external forces applied to a robot. The results exhibited excellent tracking and estimation performance of the proposed system. It is believed that the introduced scheme is fundamental and can be more developed to achieve more desired properties. For example, it can be made robust to unstructured uncertainties (due to adaptation, structured uncertainties are handled automatically).

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References


Appendix

Theorem 1 [26, p.171]: Let \( f_i \) for \( i = 1, 2, \ldots, n \) be a \( 1 \times p \) complex-valued continuous functions defined on \([t_1, t_2]\). Let \( F \) be the \( n \times p \) matrix with \( f_i \) as its ith row. Define

\[
W(t_1, t_2) = \int_{t_1}^{t_2} F(t)F^*(t) \, dt
\]
Then $f_1, f_2, ..., f_n$ are linearly independent on $[t_1, t_2]$ if and only if the $n \times n$ constant matrix $W(t_1, t_2)$ is nonsingular.

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